Numeric Response Questions

Tangents and Normal

Q.1 If the line joining the points (0,3) and $(5,-2)$ is a tangent to the curve $y=\frac{c}{x+1}$, then find the value of c.

Q.2 Find the number of points to the curve $x^{25}+y^{2t1}=a^{351}$ where the tangents are equally inclined to the axes.

Q.3 Find the length of the subtangent to the curve $x^2+xy+y^2=7$ at $(1,-3)$.

Q.4 A particle moves along the curve $y=x^{32}$ in the furst quadrant in such a way that its distance from the origin increase at the rate of 11 units per second. Then find the value of $\frac{dx}{dt}$ at $x=3$.

Q.5 Let $f(\mathrm{x})=\mathrm{x}^3+\mathrm{ax}+\mathrm{b}$ with $\mathrm{a}\neq\mathrm{b}$ and suppose the tangent lines to the graph of f at $\mathrm{x}=\mathrm{a}\&\mathrm{x}=\mathrm{b}$ have the same gradient. Then find the value of $f(1)$.

Q.6 If the curves $\frac{x^2}{a^2}+\frac{y^2}{4}=1$ and $y^3=16x$ intersect at right angle; then find value of a^2.

Q.7 Find the length of the sub-tangent to the curve $\sqrt{x}+\sqrt{y}=3$ at the point (4,1).

Q.8 The tangent to the curve $2y^3=ax^2+x^3$ at (a,a) cuts of intercepts p and q on co-ordinate axes. If $p^2+q^2=61$, then find value of a

Q.9 Find the area of the triangle formed by the tangent to the curve $\sin y=x^3-x^5$ at the point (1, 0) and the coordinate axes.

Q.10 If tangent of acute angle between the curves $y=|x^2-1|$ and $y=\sqrt{7-x^2}$ at their points of intersection is $k\sqrt{3}$, then find k ?

Q.11 If Lagrange's mean value theorem is applicable for the function $f(x)=\begin{cases} mx+c, & x<0 \\ e^x, & x\geq 0\end{cases}$ in $[-2,2]$ then find the value of $m+3c$.

Q.12 If $f(x)=x(x-2)(x-4), 1\leq x\leq 4$, then find a number satisfying the conditions of the mean value theorem.

Q.13 Water is dripping out from a conical funnel of semi-vertical angle $\frac{\pi}{4}$ at the uniform rate of $2\ \mathrm{cm}^{3/sec}$ in its surface area through a tiny hole at the vertex in the bottom. When the slant height of

Get More Learning Materials Here : CLICK HERE www.studentbro.in

the water is 4 cm, if the rate of decrease of the slant height of the water, is $\frac{\sqrt{k}}{4\pi}$ cm/sec then find k.

Q.14 If $4x^2 + py^2 = 45$ and $x^2 - 4y^2 = 5$ cut orthogonally, then find the value of p.

Q.15 Side of an equilateral triangle expands at the rate of 2 cm/sec. If the rate of increase of its area when each side is 10 cm is $k\sqrt{3}$ cm^2/ sec then find k.

Get More Learning Materials Here : CLICK HERE www.studentbro.in

ANSWER KEY

1. 4.00 **2.** 4.00 **3.** 15.00 **4.** 4.00 **5.** 1.00 **6.** 1.33 **7.** 2.00
8. 30.00 **9.** 1.00 **10.** 1.25 **11.** 4.00 **12.** 1.00 **13.** 2.00 **14.** 9.00
15. 10.00

Hints & Solutions

1. line joining (0, 3) (5, – 2)

$y - 3 = \frac{-2-3}{5-0}(x - 0)$

$y - 3 = -x$

$y = 3 - x$

touches $y = \frac{c}{x+1}$

$(3 - x) = \frac{c}{x+1}$

$3x + 3 - x^2 - x = c$

$x^2 - 2x + (c - 3) = 0$

$D = 0$

$(-2)^2 - 4 \times 1\,(c - 3) = 0$

$4 - 4c + 12 = 0$

$c = 4$

3. Given curve is $x^2 + xy + y^2 = 7$

On differentiating, we get

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,-3)} = \frac{-(2-3)}{(1-6)} = -\frac{1}{5}$

Length of subtangent $= \frac{y}{\frac{dy}{dx}} = \frac{-3}{-\frac{1}{5}} = 15$

4. $r^2 = x^2 + y^2$; $r\frac{dr}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$

where $x = 3$; $y = 3\sqrt{3}$ and $r = 6$;

hence $6 \cdot 11 = 3\frac{dx}{dt} + 3\sqrt{3}\frac{dy}{dt}$

but $\frac{dy}{dt} = \frac{3}{2}\sqrt{x}\quad \frac{dx}{dt} = \frac{3\sqrt{3}}{2}\frac{dx}{dt}$

5. $f'(a) = f'(b) \Rightarrow a = -b$

So $f(1) = 1 + a + b = 1$

6. $\frac{x^2}{a^2} + \frac{y^2}{y} = 1 \quad \& \quad y^3 = 16x$

Let point of intersection is $(x_1 y_1)$

$\therefore \frac{x_1^2}{a^2} + \frac{y_1^2}{4} = 1$ ....(i)

$y_1^3 = 16x_1$ .....(ii)

Now

$\frac{2x}{a^2} + \frac{2y}{4}\frac{dy}{dx} = 0 \qquad 3y^2\frac{dy}{dx} = 16$

$\Rightarrow \frac{dy}{dx} = -\frac{4x}{a^2 y} \quad \frac{dy}{dx} = \frac{16}{3y^2}$

$m_1 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = -\frac{4x_1}{a^2 y_1}$

$m_2 = \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = \frac{16}{3y_1^2}$

$\because \theta = 90$

$\therefore m_1 m_2 = -1$

$\Rightarrow \frac{-4x_1}{a^2 y_1} \times \frac{16}{3y_1^2} = -1 \Rightarrow \frac{4x_1 \times 16}{3a^2 y_1^3} = 1$

$\Rightarrow \frac{4x_1 \times 16}{3a^2 \times 16x_1} = 1$ (from (ii))

$\Rightarrow a^2 = 4/3$

7. Given curve is

$\sqrt{x} + \sqrt{y} = 3$

Diff., w.r.t. y

$\frac{1}{2\sqrt{x}} \cdot \frac{dx}{dy} + \frac{1}{2\sqrt{y}} = 0$

$\Rightarrow \frac{dx}{dy} = -\frac{\sqrt{x}}{\sqrt{y}}$

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Length of sub-tangent $= \left|y\frac{dx}{dy}\right|$

$= \left|1\times\left(-\frac{2}{1}\right)\right|$

$= 2$

8. Given curve is $2y^3 = ax^2 + x^3$

Differentiating w.r.t. 'x'

$6y^2 \frac{dy}{dx} = 3x^2 + 2ax$

At (a, a), $6a^2 \frac{dy}{dx} = 3a^2 + 2a^2$

$\Rightarrow \frac{dy}{dx} = \frac{5}{6}$

Equation of tangent at (a, a)

$y - a = \frac{5}{6}(x - a)$

then $P = -\frac{a}{5}$, $q = \frac{a}{6}$

As, $\frac{a^2}{25} + \frac{a^2}{36} = 61$

$\Rightarrow 61a^2 = 61 \times 25 \times 36$

$\Rightarrow a^2 = 25 \times 36$

$\Rightarrow a = 5 \times 6 = 30$

9. $\sin y = x^3 - x^5$

$(\cos y)\frac{dy}{dx} = 3x^2 - 5x^4$

$\Rightarrow \frac{dy}{dx} = \frac{3x^2 - 5x^4}{\cos y}$

$\left(\frac{dy}{dx}\right)_{(1,0)} = \frac{3-5}{1} = -2$

Equation of tangent at (1, 0)

$y - 0 = -2(x - 1)$

$\Rightarrow 2x + y = 2$

$\Rightarrow \frac{x}{1} + \frac{y}{2} = 1$

Area $= \frac{1}{2}(1)(2)$

10. Solving, $|x^2 - 1| = \sqrt{7 - x^2}$

$\Rightarrow (x^2 - 3)(x^2 + 2) = 0$

$x = \pm\sqrt{3}$, so points $(\pm\sqrt{3}, 2)$

Now, $y = x^2 - 1 \Rightarrow \left(\frac{dy}{dx}\right) = 2x = 2\sqrt{3}$

$y = \sqrt{7 - x^2} \Rightarrow \frac{dy}{dx} = -\frac{x}{y} = \frac{-\sqrt{3}}{2}$

$\tan\theta = \left|\frac{5\sqrt{3}}{4}\right|$.

11. $f(0^+) = f(0^-) \Rightarrow c = 1$

$f'(0^+) = f'(0^-) \Rightarrow m = 1 \Rightarrow m + 3c = 4$

12. Apply $f'(c) = \frac{f(b) - f(a)}{b - a}$

$\Rightarrow 3c^2 - 12c + 8 = \frac{0 - 3}{3}$

$\Rightarrow 3c^2 - 12c + 9 = 0$

$\Rightarrow c^2 - 4c + 3 = 0$

$\Rightarrow (c - 1)(c - 3) = 0$

$c = 1, 3$

$\because 1 \notin (1, 4)$

13. $V = \frac{1}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \pi r^2 \frac{dr}{dt} \Rightarrow 2 = \pi r^2 \frac{dr}{dt}$

$\ell = \sqrt{2}\, r$

$\frac{d\ell}{dt} = \sqrt{2}\frac{dr}{dt} = \sqrt{2}\frac{2}{\pi(2\sqrt{2})^2} = \frac{\sqrt{2}}{4\pi}$ cm/sec.

14. Let point of intersection is (x_1, y_1)

So $4x_1^2 + py_1^2 = 45$ and $x_1^2 - 4y_1^2 = 5$

So $\frac{x_1^2}{y_1^2} = \frac{p+36}{5}$

$m_1 = -\frac{4x_1}{py_1}$ and $m_2 = \frac{x_1}{4y_1}$

$m_1 m_2 = -1 \Rightarrow -\frac{4x_1^2}{4py_1^2} = -1$

$\Rightarrow \frac{1}{P}\left(\frac{P+36}{5}\right) = 1$

$\Rightarrow P + 36 = 5p \Rightarrow p = 9$

15.[2] If x is a side of equilateral triangle

then area (A) $= \frac{\sqrt{3}}{4}x^2$

$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \times 2x \frac{dx}{dt}$

$= \frac{\sqrt{3}}{2} \times 10 \times 2$

$= 10\sqrt{3}$ cm²/sec

Get More Learning Materials Here : CLICK HERE www.studentbro.in